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**RADIATIVELY GENERATED FERMION MASSES
IN $SU(4) \times SU(2)_L \times SU(2)_R$**

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Abstract

We propose a model based on the gauge group $SU(4) \times SU(2)_L \times SU(2)_R$ where the Dirac masses of all the known fermions are generated as one-loop radiative corrections. We are able to generate realistic quark and lepton masses and mixings without a large hierarchy of Yukawa couplings or extra symmetries. The neutrino masses, which are see-saw suppressed, lie in the mass range favored to solve the solar neutrino problem. The importance of threshold corrections to tree-level mass relations in certain non-supersymmetric GUTs is demonstrated.

I. Introduction

Nature displays an obvious symmetry between leptons and quarks. To state a few of the more prominent ones: they both come in three families, the negatively charged members of corresponding quark and lepton families are similar in mass, and there is a large hierarchy between the masses of the different families. The Standard Model can account for all of these features: we understand from gauge anomaly cancellation that the number of quark and lepton families has to match up, and the fermion masses and their hierarchies are accounted for by a hierarchy of Yukawa couplings of the quarks and leptons to the Standard Higgs. However, the features we mention seem to ask for a unified explanation rather than just a way to account for them.

Another prominent feature of the fermion mass spectrum is that, compared with the other fermions in the same family, the neutrinos are practically massless. Although in the minimal Standard Model the neutrinos are strictly massless, the on-going solar and atmospheric neutrino experiments provide growing evidence for small, but non-zero neutrino mass [1, 2]. Certain cosmological models also prefer a non-zero neutrino mass [3]. A physically appealing explanation for the lightness of the neutrinos is the see-saw mechanism [4], which requires the neutrinos to be Majorana particles. Thus the neutrinos differ in a fundamental way from the rest of the fermions, all of which carry conserved electromagnetic charge, and have to be Dirac particles.

Although there has been a host of recent work on predictions for the fermion masses based on unified theories [5, 6, 7], these models usually rely on family symmetries to compute the Yukawa couplings as functions of family charge and scale of family symmetry breaking. We would like to build an extension of the Standard Model that can account for all the features we mentioned, while avoiding the extra symmetries and scales of the models of Ref. [5, 6, 7]. The model we propose unifies the quarks and leptons, doesn't have a large hierarchy of Yukawa couplings, and singles out the neutrinos as special. The model has only two scales, the unification scale v_R and the electro-weak scale v_L , with $v_R \gg v_L$. Whatever mechanism is responsible for this hierarchy in scales (manifested in our present understanding as a finely-tuned Higgs potential) would also be partially responsible for the hierarchies in fermion masses like $m_t \gg m_b, m_u$. It is not our goal to predict the fermion masses and mixings, but rather to show that one can account for them in a simple model, without a proliferation of extra symmetries and hierarchies.

A simple way to incorporate these ideas is to extend the Standard Model gauge group $SU(3) \times SU(2)_L \times U(1)_Y$ [8, 9] to the Pati-Salam group $SU(4) \times SU(2)_L \times SU(2)_R$ [10, 11], and have the fermion masses generated as one-loop radiative cor-

reactions [12]. The usual fermions transform under the gauge group as $(\underline{4}, \underline{2}, \underline{1})$ or $(\underline{4}, \underline{1}, \underline{2})$, and we introduce an extra sterile neutrino s_0 per generation. We choose the Higgses to also transform as either $(\underline{4}, \underline{2}, \underline{1})$ or $(\underline{4}, \underline{1}, \underline{2})$, which is a particularly simple and attractive choice of Higgs representation. When the neutral component of these Higgs get vacuum expectation values the gauge group is broken in the required way, but none of the fermions can get a tree-level Dirac mass. The sterile neutrino s_0 has a bare mass $m_0 \sim v_R$, and it not only acts as a mass seed in generating one-loop radiative masses for all the fermions, but also acts as the see-saw partner of the left handed neutrino, suppressing its mass relative to that of the other fermions.

This mechanism of mass generation demonstrates the impact of threshold corrections to tree level mass relations in certain GUTs. We generate fermion mass operators upon integrating out the heavy fields in the theory. The coefficients of these operators are usually expected to be small. However, as our model shows, they can be varied enough to encompass the entire spectrum of fermion masses.

In Sec. 2 we briefly discuss generic features of models of radiative mass generation, and how our model incorporates them. Sec. 3 discusses what we call the “minimal model” and is the heart of the paper. In Sec. 4 we extend the minimal model to be able to account for all the observed fermion masses and mixings and discuss the results obtained, while Sec. 5 concludes.

II. About Radiative Masses

In any model of radiative fermion mass, the generic diagram that generates a fermion mass term at one-loop looks like Fig. (1)¹

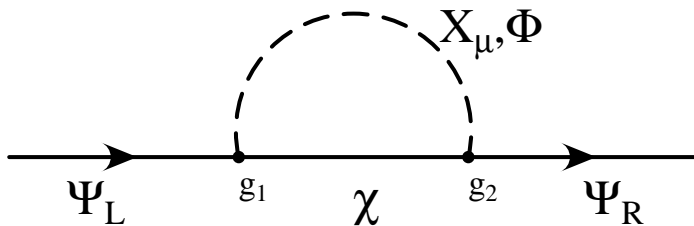


Fig. 1. The general diagram for radiative fermion mass generation.

where the external lines correspond to the standard fermions Ψ_L , Ψ_R (massless at tree-level), the dashed line could be a scalar Φ or gauge boson X_μ , and the solid internal line is a (massive) fermion χ . Since the representation content of

¹For a more detailed overview, and discussion of specific examples see Refs. [13, 14, 15].

the theory is such that there can be no tree-level mass terms connecting Ψ_L to Ψ_R , the sum of all such diagrams gives a finite and calculable contribution to the external fermion mass of order

$$m_\psi \sim \frac{g_1 g_2}{(4\pi)^2} v_L f(m_\chi/M_{X\mu,\Phi}) \quad (1)$$

where g_1 and g_2 could be gauge or Yukawa couplings, v_L is the electro-weak scale, and $f(m_\chi/M_{X\mu,\Phi})$ is some kinematical function of the masses on the internal line, with $f(m_\chi/M_{X\mu,\Phi}) \leq 1$.

In order to give the Standard Model fermions mass we need:

1) A chirality flip to turn the incoming left handed fermion into an outgoing right handed fermion. This is accomplished by having an odd number of mass insertions on the fermion line. In our case the chirality flip is provided by the bare mass of the sterile neutrino s_0 , and/or its mixings with the standard neutrinos ν_L and ν_R .

2) A change in weak isospin between left and right handed fermions. In our model this is done either by ν_L and ν_R mixing with s_0 on the fermion line, or by Higgs of different weak isospin mixing on the scalar line. These features are illustrated in Figs. (2a,2b), and are discussed in detail in the next section.

The original idea for radiative mass mechanism was to have some sort of extra symmetries so that the third generation gets mass at tree-level, the second at one-loop, and the first at two loop [16]. These models are known as the $(1, \alpha, \alpha^2)$ models. In our model, we don't impose any extra family symmetries, so all three generations get mass at one-loop, and it would fit in more closely with what are known as $(\alpha, \alpha\epsilon, \alpha\epsilon^2)$ models. (In the present case ϵ would roughly be some product of ratios of Yukawa couplings, Higgs mixing angles, and sterile neutrino masses).

One could in principle have supersymmetric models of radiative mass generation. However, in this case the formula of Eq. (1) gets modified [17] to

$$m_\psi \sim \frac{g_1 g_2}{(4\pi)^2} \frac{M_{SUSY}^2}{v_R^2} v_L f(m_\chi/M_{X\mu,\Phi}) \quad (2)$$

where M_{SUSY} is the scale of supersymmetry breaking, and v_R is the high energy scale in the problem. To get around this suppression, we need $M_{SUSY} \sim v_R$. Thus if v_R is large, then M_{SUSY} is forced to be large leaving the hierarchy problem unsolved, and if v_R is small, there is no hierarchy problem. In either case, the primary motivation for supersymmetrizing a model is lost.

III. The Minimal Model

In this section we would like to discuss a “minimal” version of the proposed model of radiative mass. This model contains only one generation of fermions, and the minimal Higgs sector needed to achieve the desired symmetry breaking. Although this minimal model is not rich enough to describe the physical world, it is extremely simple to analyze, and differs from the full-blown model in only the trivial way that the full-blown model has additional replicas of the fermion and scalar representations appearing here.

A. Representation

As mentioned earlier, the gauge group we work with is $SU(4) \times SU(2)_L \times SU(2)_R$. We write the gauge fields in the convenient matrix notation, *i.e.* we write $\hat{V}_\mu = \sum V_\mu^a \frac{\lambda^a}{2}$ where λ^a are generalized Gell-Mann matrices. In this notation, we have the following gauge fields:

$$\hat{W}_{L\mu} = \frac{1}{2} \begin{pmatrix} W_{L\mu}^0 & \sqrt{2}W_{L\mu}^+ \\ \sqrt{2}W_{L\mu}^- & -W_{L\mu}^0 \end{pmatrix} \quad (3)$$

$$\hat{W}_{R\mu} = \frac{1}{2} \begin{pmatrix} W_{R\mu}^0 & \sqrt{2}W_{R\mu}^+ \\ \sqrt{2}W_{R\mu}^- & -W_{R\mu}^0 \end{pmatrix} \quad (4)$$

$$\hat{G}_\mu = \frac{1}{2} \begin{pmatrix} G_{3\mu} + G_{8\mu}/\sqrt{3} + B_\mu/\sqrt{6} & \sqrt{2}G_{12\mu}^+ & \sqrt{2}G_{13\mu}^+ & \sqrt{2}X_{1\mu}^+ \\ \sqrt{2}G_{12\mu}^- & -G_{3\mu} + G_{8\mu}/\sqrt{3} + B_\mu/\sqrt{6} & \sqrt{2}G_{23\mu}^+ & \sqrt{2}X_{2\mu}^+ \\ \sqrt{2}G_{13\mu}^- & \sqrt{2}G_{23\mu}^- & -2G_{8\mu}/\sqrt{3} + B_\mu/\sqrt{6} & \sqrt{2}X_{3\mu}^+ \\ \sqrt{2}X_{1\mu}^- & \sqrt{2}X_{2\mu}^- & \sqrt{2}X_{3\mu}^- & -3B_\mu/\sqrt{6} \end{pmatrix}. \quad (5)$$

G_μ are the gluons, B_μ is the diagonal gauge boson that couples to $B - L$, and the X_μ are the lepto-quarks.

The fermions are in the usual representation of $SU(4) \times SU(2)_L \times SU(2)_R$ with the addition of an extra sterile neutrino s_0 ,

$$\begin{aligned} \Psi_{Li\alpha} &\sim (\underline{4}, \underline{2}, \underline{1}) \\ \Psi_{Ri\alpha} &\sim (\underline{4}, \underline{1}, \underline{2}) \\ s_0 &\sim (\underline{1}, \underline{1}, \underline{1}), \end{aligned} \quad (6)$$

where $i = 1, 2$ is the $SU(2)_L$ or $SU(2)_R$ index, and $\alpha = 1, 2, 3, 4$ is the $SU(4)$ index. Written out in matrix form, these fermion fields would look like:

$$\Psi_{L,R} = \begin{pmatrix} u_1 & u_2 & u_3 & \nu_e \\ d_1 & d_2 & d_3 & e^- \end{pmatrix}_{L,R}. \quad (7)$$

We have one “left handed” Higgs L , and one “right handed” Higgs R which transform as

$$\begin{aligned} L_{i\alpha} &\sim (\underline{4}, \underline{2}, \underline{1}) \\ R_{i\alpha} &\sim (\underline{4}, \underline{1}, \underline{2}). \end{aligned} \quad (8)$$

and in matrix form,

$$L = \begin{pmatrix} L_{u1} & L_{u2} & L_{u3} & L_\nu \\ L_{d1} & L_{d2} & R_{d3} & L_e \end{pmatrix} \quad (9)$$

and

$$R = \begin{pmatrix} R_{u1} & R_{u2} & R_{u3} & R_\nu \\ R_{d1} & R_{d2} & R_{d3} & R_e \end{pmatrix} \quad (10)$$

If we embedded this model in $SO(10)$, the fermions Ψ_L , Ψ_R would be in a $\underline{16}$ in the usual way, the Higgs L and R would also be in a $\underline{16}$, and there would be a single sterile neutrino ².

The symmetry breaking proceeds as

$$SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{\langle R_\nu \rangle} SU(3) \times SU(2)_L \times U(1)_Y \xrightarrow{\langle L_\nu \rangle} SU(3) \times U(1)_Q. \quad (11)$$

So the gauge bosons of the broken groups get tree-level masses by the usual Higgs mechanism, but the representation content makes it impossible for the standard fermions to get tree-level masses.

B. Interactions

To proceed with the analysis of the model, we break up the interaction Lagrangian into the following pieces: gauge-fermion, fermion-higgs, higgs-higgs, higgs-gauge.

i. Gauge-Fermion.

The gauge-fermion interaction is obtained from

$$i\bar{\Psi}_{L,R} \not{D}_\mu \Psi_{L,R} \quad (12)$$

where the covariant derivative is

$$D_\mu \Psi_{L,R} = \partial_\mu \Psi_{L,R} + ig_4 \hat{G}_\mu \Psi_{L,R} + ig_2 \hat{W}_{L,R\mu} \Psi_{L,R} \quad (13)$$

²A model like this has been considered in Ref. [18].

This gives us the following vertices in addition to the $SU(3)_C$, $SU(2)_L$, and $SU(2)_R$ vertices,

$$\begin{aligned}\mathcal{L}_I = & -\frac{g_4}{2\sqrt{6}}\bar{u}_L B^\mu \gamma_\mu u_L - \frac{g_4}{2\sqrt{6}}\bar{d}_L B^\mu \gamma_\mu d_L + \frac{3g_4}{2\sqrt{6}}\bar{\nu}_L B^\mu \gamma_\mu \nu_L + \frac{3g_4}{2\sqrt{6}}\bar{e}_L B^\mu \gamma_\mu e_L \\ & -\frac{g_4}{\sqrt{2}}[\bar{u}_L X^\mu \gamma_\mu \nu_L + h.c.] - \frac{g_4}{\sqrt{2}}[\bar{d}_L X^\mu \gamma_\mu e_L + h.c.] \\ & + L \leftrightarrow R.\end{aligned}\tag{14}$$

ii. Fermion-Higgs

The representation content of the model makes this sector extremely simple:

$$\mathcal{L}_Y = -\kappa_L[\Psi_L^{i\alpha} L_{i\alpha} s_0 + h.c.] - \kappa_R[\Psi_R^{i\alpha} R_{i\alpha} s_0^c + h.c.].\tag{15}$$

Written out in components, this is

$$\begin{aligned}\mathcal{L}_Y = & -\kappa_L[\bar{u}_L L_u s_0 + \bar{d}_L L_d s_0 + \bar{\nu}_L L_\nu s_0 + \bar{e}_L L_e s_0 + h.c.] \\ & -\kappa_R[\bar{u}_R R_u s_0^c + \bar{d}_R R_d s_0^c + \bar{\nu}_R R_\nu s_0^c + \bar{e}_R R_e s_0^c + h.c.].\end{aligned}\tag{16}$$

When L_ν and R_ν get vacuum expectation values $v_L/\sqrt{2}$ and $v_R/\sqrt{2}$ we get the following tree-level neutrino mass matrix

$$M_\nu = \begin{pmatrix} 0 & 0 & \kappa_L v_L/\sqrt{8} \\ 0 & 0 & \kappa_R v_R/\sqrt{8} \\ \kappa_L v_L/\sqrt{8} & \kappa_R v_R/\sqrt{8} & m_0 \end{pmatrix},\tag{17}$$

where $m_0 \sim v_R$ is the bare mass of the sterile neutrino, and we've chosen the basis (ν_L^c, ν_R, s_0) . This matrix will be diagonalized by an orthogonal matrix \hat{O} , with matrix elements O_{ij} , and will have eigenvalues $m_1 = 0$, m_2 , $m_3 \sim v_R$. So the left handed neutrino is massless at tree-level. It will get a Dirac mass at one loop like the rest of the fermions, but its physical mass will be see-saw suppressed and much smaller than that of the other fermions.

iii. Higgs-Higgs.

$$\begin{aligned}V(L, R) = & -2\mu_L^2 L_{i\alpha} L^{i\alpha} + \lambda_{L1} L_{i\alpha} L^{i\alpha} L_{j\beta} L^{j\beta} + \lambda_{L2} L_{i\alpha} L^{j\alpha} L^{i\beta} L_{j\beta} + \lambda_{L3} L_{i\alpha} L^{j\alpha} L_{\beta}^i L_j^\beta \\ & -2\mu_R^2 R_{i\alpha} R^{i\alpha} + \lambda_{R1} R_{i\alpha} R^{i\alpha} R_{j\beta} R^{j\beta} + \lambda_{R2} R_{i\alpha} R^{j\alpha} R^{i\beta} R_{j\beta} + \lambda_{R3} R_{i\alpha} R^{j\alpha} R_{\beta}^i R_j^\beta \\ & + \lambda_{LR1} L_{i\alpha} L^{i\alpha} R_{j\beta} R^{j\beta} + \lambda_{LR2} L_{i\alpha} R^{j\alpha} L^{i\beta} R_{j\beta} + \lambda_{LR3} (L_{i\alpha} R^{j\alpha} L_{\beta}^i R_j^\beta + h.c.)\end{aligned}\tag{18}$$

Here $L^{i\alpha} = (L_{i\alpha})^*$ and $L_\alpha^i = \epsilon^{ij} L_{j\alpha}$. In the limit of no L-R coupling, *i.e.* $\lambda_{LR1} = \lambda_{LR2} = \lambda_{LR3} = 0$ we can generalize the arguments of Ref. [19] to show that if the following conditions are satisfied:

$$\begin{aligned} \lambda_{L2} &< 0; \quad \lambda_{L1} + \lambda_{L2} > 0; \quad \lambda_{L3} > \lambda_{L2} \quad \text{or} \quad |\lambda_{L3}| > 2\lambda_{L1} + \lambda_{L2} \\ \lambda_{R2} &< 0; \quad \lambda_{R1} + \lambda_{R2} > 0; \quad \lambda_{R3} > \lambda_{R2} \quad \text{or} \quad |\lambda_{R3}| > 2\lambda_{R1} + \lambda_{R2} \end{aligned} \quad (19)$$

the absolute minimum of the potential is at

$$\begin{aligned} \langle L \rangle &= \begin{pmatrix} 0 & 0 & 0 & v_L/\sqrt{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \langle R \rangle &= \begin{pmatrix} 0 & 0 & 0 & v_R/\sqrt{2} \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (20)$$

This minimum will be stable to turning on the L-R couplings when

$$\lambda_{LR2} < 0; \quad \lambda_{LR1} + \lambda_{LR2} = 0; \quad \lambda_{LR3} < \lambda_{LR2}/2 \quad (21)$$

(Note the conditions on λ_{LR} are sufficient, but not necessary conditions).

The minima are then determined from the equations

$$(\lambda_{L1} + \lambda_{L2})v_L^2 + \frac{1}{2}(\lambda_{LR1} + \lambda_{LR2})v_R^2 = 2\mu_L^2 \quad (22)$$

and

$$\frac{1}{2}(\lambda_{LR1} + \lambda_{LR2})v_L^2 + (\lambda_{R1} + \lambda_{R2})v_R^2 = 2\mu_R^2. \quad (23)$$

The parameters will have to be fine tuned to generate the hierarchy $v_R \gg v_L$. There will be in general only one light neutral Higgs, with the rest of the physical Higgses having mass $\sim v_R$.

iv. Higgs-Gauge

Not much detail is required here, except that given the symmetry breaking scheme assumed, we get the following masses for the charged gauge bosons:

$$M_X^2 = \frac{g_S^2}{4}[v_R^2 + v_L^2]; \quad M_{W_L}^2 = \frac{g_L^2}{4}v_L^2; \quad M_{W_R}^2 = \frac{g_R^2}{4}v_R^2 \quad (24)$$

The neutral gauge bosons have the mass squared matrix

$$M_0 = \frac{1}{8} \begin{pmatrix} g_L^2 v_L^2 & 0 & -(3/\sqrt{6})g_L g_S v_L^2 \\ 0 & g_R^2 v_R^2 & -(3/\sqrt{6})g_R g_S v_R^2 \\ -(3/\sqrt{6})g_L g_S v_L^2 & -(3/\sqrt{6})g_R g_S v_R^2 & (3/2)g_S^2(v_R^2 + v_L^2) \end{pmatrix} \quad (25)$$

in the basis (W_L^0, W_R^0, B^0) , with eigenvalues

$$M_\gamma^2 = 0; \quad M_Z^2 = \frac{v_L^2}{4} \left[g_L^2 + \frac{3g_R^2 g_S^2}{2g_R^2 + 3g_S^2} + \mathcal{O}(v_L^2/v_R^2) \right]; \quad M_{Z'}^2 = \frac{v_R^2}{8} [2g_R^2 + 3g_S^2 + \mathcal{O}(v_L^2/v_R^2)] \quad (26)$$

and eigenvectors

$$\begin{pmatrix} \gamma \\ Z \\ Z' \end{pmatrix} = \begin{pmatrix} s_W & s_W & \sqrt{c_{2W}} \\ c_W & -t_W s_W & -t_W \sqrt{c_{2W}} \\ 0 & -\sqrt{c_{2W}}/c_W & t_W \end{pmatrix} \begin{pmatrix} W_L^0 \\ W_R^0 \\ B_0 \end{pmatrix}. \quad (27)$$

For $v_L \ll v_R$, the usual electro-weak relation $M_W^2 = M_Z^2 c_W^2$ is still maintained where we define

$$g_L^2 = \frac{e^2}{s_W^2} \Rightarrow s_W^2 = \frac{3g_R^2 g_S^2}{3g_R^2 g_S^2 + 2g_L^2 g_R^2 + 3g_L^2 g_S^2} \quad (28)$$

(All of these tree-level relations are defined at the unification scale $v_R = M_U$). For the rest of this paper we will assume that $g_L(M_U) = g_R(M_U) = g_2(M_U)$, in which case we have the relation

$$s_W^2(M_U) = \frac{1}{2} - \frac{\alpha(M_U)}{3\alpha_S(M_U)} \quad (29)$$

If we use as inputs at the electroweak scale $\alpha^{-1} = 128.5$, $s_W^2 = 0.23$, and $\alpha_S^{-1} = 8.33$, then using the one-loop β functions with only gauge boson and fermion contributions, this gives us $M_U = v_R \sim 10^{14}$ GeV, with $\alpha_S^{-1}(M_U) \sim 40$, $\alpha_L^{-1} = \alpha_R^{-1} \sim 45$.

C. Fermion Masses.

From the interaction vertices we have written down we can see that the processes that generate one-loop masses for the fermions are leptoquark gauge boson exchange (Fig. (2a)), which generates mass for the up type quarks, Higgs exchange (Fig. (2b)) which generates mass for all the fermions, and neutral gauge boson exchange (Figs. (3a,3b)) which generate Dirac and Majorana masses respectively for the neutrinos.

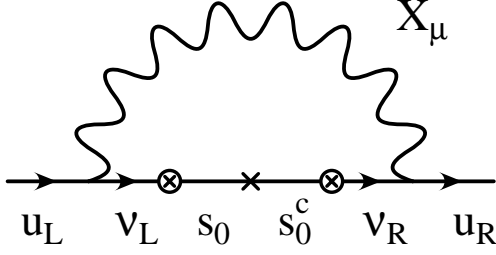


Fig. 2a. Gauge boson exchange.

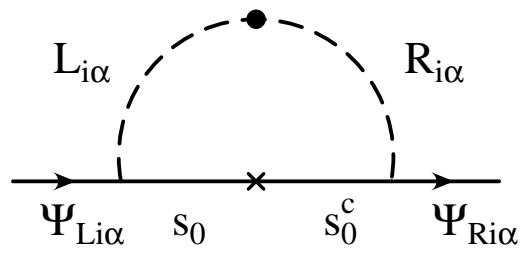


Fig. 2b. Higgs exchange.

These diagrams are drawn in the interaction basis. \times indicates a fermion mass insertion, \circ indicates a change in left or right weak isospin, and \bullet indicates a change in both left and right weak isospin.

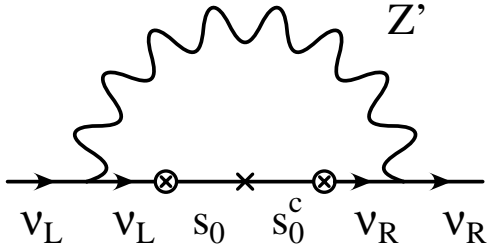


Fig. 3a Neutrino Dirac mass.

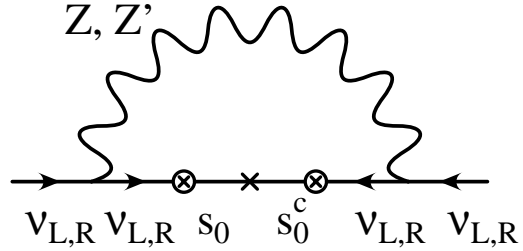


Fig. 3b Neutrino Majorana mass.

We see right away that this model gives us a way to differentiate up type masses from down type masses. The up type fermions get their masses from both gauge boson exchange and Higgs exchange, whereas the down types get theirs only from Higgs exchange.

The results for the two types of diagrams calculated in 't Hooft-Feynman gauge are

$$m_{gauge}(M_U) = 2 \frac{g_1 g_2}{(4\pi)^2} \sum_{i=1}^3 O_{xi} O_{yi} m_i \left[\frac{m_i^2}{m_i^2 - M_G^2} \ln\left(\frac{m_i^2}{M_G^2}\right) \right] \quad (30)$$

$$m_{Hj\alpha}(M_U) = \frac{\kappa_L \kappa_R}{(4\pi)^2} s_{j\alpha} c_{j\alpha} \sum_{i=1}^3 O_{3i}^2 m_i \left[\frac{M_{j\alpha 1}^2}{m_i^2 - M_{j\alpha 1}^2} \ln\left(\frac{m_i^2}{M_{j\alpha 1}^2}\right) - \frac{M_{j\alpha 2}^2}{m_i^2 - M_{j\alpha 2}^2} \ln\left(\frac{m_i^2}{M_{j\alpha 2}^2}\right) \right] \quad (31)$$

where m_i are the masses of the physical neutrinos, and O_{ij} are elements of the orthogonal matrix that diagonalizes the neutrino mass matrix (17). M_G is the

mass of the appropriate physical gauge boson, and g_1, g_2 are its couplings to the fermions. As an example, for the quarks, $g_1 = g_2 = g_s$, and $M_G = M_X$. The subscripts x,y on the matrix O depend on the basis (ν_L^c, ν_R, s_0) of Eq. (17). For Dirac masses x=1, y=2; left-handed Majorana mass x=1, y=1; right-handed Majorana mass x=2, y=2. $M_{j\alpha 1,2}$ are the eigenvalues of the Higgs mass matrix³ in the basis $(L_{j\alpha}, R_{j\alpha})$, and $s_{j\alpha}$ and $c_{j\alpha}$ are the sine and cosine of the mixing angles between $L_{j\alpha}$ and $R_{j\alpha}$. Explicit examples of Higgs mass matrices are presented in the appendix (Eqs. (64-72)). We would like to emphasize that Eqs. (30,31) are valid at the scale M_U . In order to compare with the known fermion masses we need to use the renormalization group to run the masses down to the appropriate scale. Eqs. (30,31) are the basis of all mass calculations in this paper, and we would like to first discuss some general features, then illustrate their use with some examples.

The product $O_{1i}O_{2i}$, valid for Dirac masses, in Eq. (30) represents mixing between ν_L and ν_R . One can tell by inspection of the mass matrix Eq. (17) that it is of order v_L/v_R , while the masses m_i and M_X are of order v_R . Increasing the mass m_i while holding v_L fixed simply decreases the product $O_{1i}O_{2i}$. This is just a manifestation of the decoupling theorem: one cannot generate arbitrarily large one-loop masses m_{gauge} by increasing m_i as a naive inspection of Eq. (30) seems to show. In fact in order to maximize m_{gauge} the masses m_i are constrained to lie close to the unification scale M_U . A numerical study shows that the maximum $m_{gauge}(M_U)$ can be is ~ 10 GeV.

In order to ensure the reliability of perturbative calculations, we impose the following constraints on the Yukawa and Higgs couplings:

$$\frac{\kappa^2}{4\pi} < 1 \Rightarrow \kappa < 3.5; \quad M_h^2 < 4v_R^2 \Rightarrow \lambda_i < 4. \quad (32)$$

The Higgs mixing angle $s_{j\alpha}$ which represents mixing between $L_{j\alpha}$ and $R_{j\alpha}$ is generically of order v_L/v_R . One can in general fine tune the Higgs potential to increase the mixing angle. However, a result of this fine tuning is that the difference $(M_{j\alpha 1} - M_{j\alpha 2}) \rightarrow 0$, and the two terms in Eq. (31) interfere destructively (this is illustrated in the appendix). Thus there is an upper bound to $m_{Higgs}(M_U)$ which is also around 10 GeV.

One can also see that the up type quarks get contributions from both gauge boson and Higgs exchange, while the down type quarks get a contribution only from Higgs exchange. This feature will be important in the full-blown model, as interference between different diagrams will either enhance or suppress the up type quark masses compared with the masses of the down type quarks.

³Unless some of the Higgs are Nambu-Goldstone bosons, in which case one uses the mass of the corresponding gauge boson.

Let us now consider an example:

If we choose as inputs

$$\kappa_L = \kappa_R = 0.6; \quad m_0 = 0.4v_R; \quad (33)$$

$$\begin{aligned} M_{u1}^2 &= 0.05 v_R^2 & M_{u2}^2 &= M_X^2 & s_u &= \epsilon \\ M_{d1}^2 &= 0.05 v_R^2 & M_{d2}^2 &= 0.025 v_R^2 & s_d &= 6 \epsilon \end{aligned} \quad (34)$$

where $\epsilon = v_L/v_R$, we get

$$m_c(100 \text{ GeV}) = 900 \text{ MeV}; \quad m_s(100 \text{ GeV}) = 100 \text{ MeV}. \quad (35)$$

If we choose

$$\kappa_L = \kappa_R = 0.04; \quad m_0 = 0.04v_R; \quad (36)$$

$$\begin{aligned} M_{u1}^2 &= 0.05 v_R^2 & M_{u2}^2 &= M_X^2 & s_u &= \epsilon \\ M_{d1}^2 &= 0.05 v_R^2 & M_{d2}^2 &= 0.025 v_R^2 & s_d &= 80 \epsilon \end{aligned} \quad (37)$$

we get

$$m_u(100 \text{ GeV}) = 5 \text{ MeV}; \quad m_d(100 \text{ GeV}) = 6 \text{ MeV}. \quad (38)$$

All the Higgs masses and mixings were evaluated from the Higgs potential of Eq. (18), incorporating the constraints of Eqs. (21-23, 32). This example serves to illustrate how a large hierarchy between the charm and up masses is generated by several smaller hierarchies in coupling constants and masses, and also that it is possible to generate a large hierarchy in the up sector and simultaneously a smaller hierarchy in the down sector by tuning the Higgs parameters. Of course we are not free to use different Higgs couplings for the different generations as we have here. We use it here for illustrative purposes, as when we enlarge the Higgs sector, a similar effect does occur. Relative sizes of different Higgs self couplings determine the sign and magnitude of the Higgs mixing angles, which in turn determine whether different diagrams interfere constructively or destructively.

An obvious problem with this model is that even with two families, the couplings are all diagonal, and we never get inter-family mixing. Another problem with this minimal model is that, as the arguments about upper bounds on the masses show, we cannot generate a realistic mass for the top quark. Finally, this model cannot generate any mass at all for the charged leptons. This is because the Higgs L_e and R_e are exact Nambu-Goldstone bosons (they are eaten by the W_L and W_R) and do not mix at tree-level.

Thus we see that although the “minimal” model serves as an attractive and educative example of fermion mass generation, it is not rich enough to describe the physical world. However, if one simply extends the Higgs sector to include another pair of Higgs Λ and T that transform exactly like L and R , all these

problems disappear, and one can in fact generate realistic fermion masses and mixings. This is the subject of the next section.

IV. The Full-Blown Model

In this section we would like to extend the minimal scenario of the previous section, and present a complete model of fermion masses and mixing. In order to accomplish this we first introduce another pair of Higgs $\Lambda \sim (\underline{4}, \underline{2}, \underline{1})$, and $T \sim (\underline{4}, \underline{1}, \underline{2})$. This simple addition to the model greatly complicates the Higgs potential, and we relegate a detailed discussion to the appendix. We should point out, however, that in order for this model to give the charged leptons mass, and still accomplish the correct gauge symmetry breaking, we must have the vacuum expectation values $\langle \Lambda \rangle = \langle T \rangle = 0$. Thus the low energy data selects out a particular region in the space of possible Higgs couplings (this is in some sense analogous to the results in supersymmetric models of fermion mass where the large $\tan \beta$ case seems to be preferred by the low energy data [5]). As a result of this particular choice of vacuum expectation values, the formulas (24,26) for the gauge boson masses remain unchanged. This also precludes the possibility of introducing CP violation in the Higgs sector.

Next we generalize the Yukawa coupling constants to matrices of couplings. The Higgs-fermion interaction Eq. (16) now looks like

$$\begin{aligned} \mathcal{L}_Y = & -\kappa_L^{ab} [\Psi_{La}^{j\alpha} L_{j\alpha} s_{0b} + h.c.] - \kappa_R^{ab} [\Psi_{Rb}^{j\alpha} R_{j\alpha} s_{0a}^c + h.c.] \\ & -\kappa_\Lambda^{ab} [\Psi_{La}^{j\alpha} \Lambda_{j\alpha} s_{0b} + h.c.] - \kappa_T^{ab} [\Psi_{Rb}^{j\alpha} T_{j\alpha} s_{0a}^c + h.c.], \end{aligned} \quad (39)$$

where a, b are family indices. The neutrino mass matrix, and the orthogonal matrix \hat{O} that diagonalizes it, will now be 9×9 matrices. The basis we will use in our subsequent discussion groups all the left handed neutrinos first, followed by the right handed, and finally the sterile neutrinos *i.e.* $(\nu_{Le}^c, \nu_{L\mu}^c, \nu_{L\tau}^c, \nu_{Re}, \nu_{R\mu}, \nu_{R\tau}, s_{0e}, s_{0\mu}, s_{0\tau})$.

The gauge-fermion interaction will still be diagonal, with Eq. (12) generalized to include diagonal family indices.

Finally, we come to the formula for fermion masses in the model. Most of the work has already been done in the previous section, and all we need to do is generalize Eqs. (30,31) to sum over different diagrams, and account for inter-family mixing.

$$m_{gauge}^{ab}(M_U) = 2 \frac{g_1 g_2}{(4\pi)^2} \sum_{i=1}^9 O_{x,i} O_{y,i} m_i \left[\frac{m_i^2}{m_i^2 - M_G^2} \ln \left(\frac{m_i^2}{M_G^2} \right) \right]. \quad (40)$$

Here $x=a, y=b+3$ for Dirac masses; $x=a, y=b$ for left handed Majorana masses;

$x=a+3, y=b+3$ for right handed Majorana masses.

$$m_{LR,j\alpha}^{ab}(M_U) = \sum_{m,n=1}^3 \frac{\kappa_L^{am} \kappa_R^{bn}}{(4\pi)^2} s_{LRj\alpha} c_{LRj\alpha} \sum_{i=1}^9 O_{m+6,i} O_{n+6,i} m_i$$

$$\left[\frac{M_{LRj\alpha 1}^2}{m_i^2 - M_{LRj\alpha 1}^2} \ln\left(\frac{m_i^2}{M_{LRj\alpha 1}^2}\right) - \frac{M_{LRj\alpha 2}^2}{m_i^2 - M_{LRj\alpha 2}^2} \ln\left(\frac{m_i^2}{M_{LRj\alpha 2}^2}\right) \right] \quad (41)$$

$$m_{\Lambda T,j\alpha}^{ab}(M_U) = \sum_{m,n=1}^3 \frac{\kappa_\Lambda^{am} \kappa_T^{bn}}{(4\pi)^2} s_{\Lambda Tj\alpha} c_{\Lambda Tj\alpha} \sum_{i=1}^9 O_{m+6,i} O_{n+6,i} m_i$$

$$\left[\frac{M_{\Lambda Tj\alpha 1}^2}{m_i^2 - M_{\Lambda Tj\alpha 1}^2} \ln\left(\frac{m_i^2}{M_{\Lambda Tj\alpha 1}^2}\right) - \frac{M_{\Lambda Tj\alpha 2}^2}{m_i^2 - M_{\Lambda Tj\alpha 2}^2} \ln\left(\frac{m_i^2}{M_{\Lambda Tj\alpha 2}^2}\right) \right] \quad (42)$$

We would like to illustrate with an explicit example, that the model can indeed generate realistic fermion mass matrices via Eqs. (40,41,42). Our constraints are to not have a large hierarchy of coupling constants, to keep coupling constants below a magnitude where we can trust perturbation theory, and finally to work honestly from the Lagrangian of the model. This last point essentially states that we have to keep in mind the relation between the Higgs masses and mixing angles: they are not independent. As we show in the appendix, the mixing angle can be made large only at the expense of lowering the differences in the Higgs masses. This is a fact that is often not explicitly accounted for in papers on radiative mass generation.

Let us take the following as inputs to the model (all defined at the scale $v_R \sim 10^{14}$ GeV).

i) Yukawa couplings.

$$\kappa_L = \begin{pmatrix} 0.04 & 0.03 & 0.06 \\ 0.06 & 0.42 & 0.24 \\ 0.06 & 0.08 & 3.5 \end{pmatrix}; \quad \kappa_R = \begin{pmatrix} 0.04 & 0.03 & 0.06 \\ 0.06 & 0.42 & 0.24 \\ 0.06 & 0.08 & 3.5 \end{pmatrix}$$

$$\kappa_\Lambda = \begin{pmatrix} -0.2 & -0.12 & -0.2 \\ -0.26 & -1.2 & 0.28 \\ 3.5 & 3.5 & 3.5 \end{pmatrix}; \quad \kappa_T = \begin{pmatrix} 0.2 & 0.1 & 0.2 \\ -0.34 & 0.8 & 0.18 \\ 3.5 & 3.5 & 3.5 \end{pmatrix} \quad (43)$$

ii). Sterile neutrino bare masses.

$$m_{s_e} = 0.5 \, v_R; \quad m_{s_\mu} = 1.0 \, v_R; \quad m_{s_\tau} = 7.0 \, v_R. \quad (44)$$

iii). Higgs vacuum expectation values, masses, and mixings.

$$v_L = 290 \, \text{GeV}; \quad v_R = 10^{14} \, \text{GeV}; \quad \epsilon = \frac{v_L}{v_R}. \quad (45)$$

$$\begin{aligned}
M_{LRu1}^2 &= 2.0 \, v_R^2 & M_{LRu2}^{2'} &= M_X^2 & s_{LRu} &= \epsilon \\
M_{\Lambda Tu1}^2 &= 2.0 \, v_R^2 & M_{\Lambda Tu2}^2 &= 0.25 \, v_R^2 & s_{\Lambda Tu} &= 1.14\epsilon \\
M_{LRd1}^2 &= 2.0 \, v_R^2 & M_{LRd2}^2 &= 0.5 \, v_R^2 & s_{LRd} &= -0.03\epsilon \\
M_{\Lambda Td1}^2 &= 2.0 \, v_R^2 & M_{\Lambda Td2}^2 &= 0.40 \, v_R^2 & s_{\Lambda Td} &= 0.05\epsilon \\
M_{LRe1}^{2'} &= M_{W_R}^2 & M_{LRe2}^{2'} &= M_{W_L}^2 & s_{LRe} &= 0 \\
M_{\Lambda Te1}^2 &= 4.0 \, v_R^2 & M_{\Lambda Te2}^2 &= 1.0 \, v_R^2 & s_{\Lambda Te} &= 0.08\epsilon \\
M_{LR\rho1}^2 &= M_{Z'}^2 & M_{LR\rho2}^2 &= M_Z^2 & s_{LR\rho} &= 0 \\
M_{LR\eta1}^{2'} &= M_{Z'}^2 & M_{LR\eta2}^{2'} &= M_Z^2 & s_{LR\eta} &= 0 \\
M_{\Lambda T\rho1}^2 &= 3.8 \, v_R^2 & M_{\Lambda T\rho2}^2 &= 3.0 \, v_R^2 & s_{\Lambda T\rho} &= 0.3\epsilon \\
M_{\Lambda T\eta1}^2 &= 3.8 \, v_R^2 & M_{\Lambda T\eta2}^2 &= 3.0 \, v_R^2 & s_{\Lambda T\eta} &= -0.3\epsilon
\end{aligned} \tag{46}$$

All of these Higgs parameters are derived from the Higgs potential (Eq. (56)), keeping in mind the extremization equations (Eqs. (58-61)), and the constraints on the sizes of the couplings (Eq. (32)). The mass values that are primed correspond to the Nambu-Goldstone bosons, and the masses of the corresponding gauge bosons are given in Eqs. (24,26). These masses, and the mixing angles for the Nambu-Goldstone bosons are set by Eqs. (58-61) and cannot be adjusted. Notice that we have chosen $v_L(v_R) = 290 \, \text{GeV}$ to account for its running. Given this choice of inputs we get the following outputs at the electroweak scale ($\sim 100 \, \text{GeV}$)

$$m_u = 1 \, \text{MeV}; \quad m_c = 1 \, \text{GeV}; \quad m_t = 180 \, \text{GeV}. \tag{47}$$

$$m_d = 2 \, \text{MeV}; \quad m_s = 100 \, \text{MeV}; \quad m_b = 4 \, \text{GeV}. \tag{48}$$

The absolute values of the quark mixing matrix are

$$|V_{KM}| = \begin{pmatrix} 0.98 & 0.2 & 0.05 \\ 0.2 & 0.98 & 0.1 \\ 0.05 & 0.1 & 0.99 \end{pmatrix} \tag{49}$$

$$m_e = 0.1 \, \text{MeV}; \quad m_\mu = 60 \, \text{MeV}; \quad m_\tau = 3 \, \text{GeV}. \tag{50}$$

$$m_{\nu_e} = 1 \times 10^{-4} \, \text{eV}; \quad m_{\nu_\mu} = 3 \times 10^{-3} \, \text{eV}; \quad m_{\nu_\tau} = 4 \times 10^{-2} \, \text{eV}. \tag{51}$$

The absolute values of the lepton mixing matrix are

$$|V_\nu| = \begin{pmatrix} 0.99 & 0.1 & 0.05 \\ 0.1 & 0.98 & 0.15 \\ 0.05 & 0.15 & 0.99 \end{pmatrix} \tag{52}$$

As an example, let us explore how the top-bottom mass hierarchy arises in this model. The top quark gets an $\mathcal{O}(10) \, \text{GeV}$ contribution from gauge boson exchange, $\mathcal{O}(10) \, \text{GeV}$ contributions from each of the 3 diagrams involving $L - R$

Higgs exchange, and additional $\mathcal{O}(40)$ GeV contributions from the 3 diagrams with $\Lambda - T$ Higgs exchange for a total mass of $\mathcal{O}(160)$ GeV. On the other hand, the bottom quark gets no contribution from gauge boson exchange, a total $\mathcal{O}(5)$ GeV from $L - R$ Higgs exchange, and an *opposing* $\mathcal{O}(10)$ GeV from $\Lambda - T$ exchange for a total of $\mathcal{O}(5)$ GeV. The exact expressions for the Higgs masses and mixings are in Eqs. (64-72) of the appendix.

We would like to discuss what we have accomplished. We started with a well motivated extension of the Standard Model, the Pati-Salam group $SU(4) \times SU(2)_L \times SU(2)_R$. We add one sterile neutrino per generation to the Standard Model fermion content. The Higgs sector is extremely simple, all Higgs fields transforming as $(\underline{4}, \underline{2}, \underline{1})$ or $(\underline{4}, \underline{1}, \underline{2})$. The magnitude of every coupling constant in the theory lies within the range

$$0.03 < g_i, \kappa_i, \lambda_i < 4, \quad (53)$$

which is a hierarchy of ~ 100 (actually it's only the Yukawa couplings that saturate this hierarchy; the Higgs couplings are all within a factor of 10 of each other). Certain Higgs self couplings are fine-tuned to give the symmetry breaking we want, while the rest are assigned values within the range of Eq. (53). The fermions receive masses as one-loop radiative corrections.

All fermions within a generation have *identical* Yukawa couplings to the Higgs (39), but the Higgs to which the different fermions couple may have different masses and mixings depending on the Higgs potential. Using this fact, as well as the fact that the up type quarks get their mass from gauge boson exchange in addition to Higgs exchange, this model can generate the large hierarchy $m_u/m_t \sim 10^{-5}$ while simultaneously generating the much smaller hierarchy $m_d/m_b \sim 10^{-3}$. The up-down mass inversion ($m_u < m_d$) is also achieved. The elements of the quark mixing matrix have the correct order of magnitude as well as hierarchies. Although the magnitudes of V_{ub} and V_{cb} are larger than the experiment numbers, we postpone a search for more realistic values till we have incorporated CP violation into this model.

The charged leptons have realistic masses, and the neutrino masses and lepton mixing matrix have interesting values. For $\nu_e - \nu_\mu$ we have $\delta m^2 \sim 10^{-5} eV^2$, which lies in the range favored by the MSW [20] solution to the solar neutrino problem [1]. These neutrino masses are fairly robust since most of the neutrino mass comes from gauge boson exchange (this is a natural consequence of the fact that the masses and mixings of the neutral Higgses are constrained by their role in the spontaneous symmetry breaking), however the mixing angle is dominated by the charged lepton mixing, and dependent on the Yukawa couplings.

We think it to be extremely non-trivial, that such a simple extension of the Standard Model can account for all of these features. However, since we restricted

ourselves to real masses and coupling constants, there is no CP violation in this model. It is our hope that we can generalize this model to include sources of CP violation that can not only account for the observed low energy CP violation in the K meson system, but also the observed baryon asymmetry. We are currently pursuing this possibility, as well as studying possible low energy signatures of this model [21].

Although at this stage our aim is not to precisely reproduce the known values for all of the observables (Eqs. (47-52)), we should point out our approximations in obtaining them. Eqs. (40-42) were evaluated using the inputs of Eqs. (43-46) to obtain mass matrices at the scale v_R . The mass matrices were numerically diagonalized, giving the mass eigenvalues and mixing matrices also at the scale v_R . In our approximation, none of the mixing angles or the lepton masses vary with scale. Each individual quark mass eigenvalue was scaled using the one-loop β functions with only gauge boson and fermion contributions. Effectively this corresponded to an enhancement of ~ 2.6 in each quark mass between the scales $v_R \sim 10^{14}$ GeV and $M_W \sim 100$ GeV. We estimate these approximations to introduce errors in the quark masses of order 20% – 30%, in the lepton masses to be $\sim 5\% - 10\%$, and in the mixing angles $\sim 5\%$.

In a sense one could think of this model as demonstrating the importance of threshold corrections to tree-level mass relations in certain non-supersymmetric GUTs. We have started with the tree level mass relation $m_{lepton} = m_{down} = m_{up} = 0$, and essentially generated the entire fermion mass spectrum as a consequence of the matching conditions when we integrate out the heavy fields! One could also envision the model we have presented as being an intermediate scale effective theory of an $SO(10)$ GUT, with sterile neutrinos and Higgs in the 16, as in Ref. [18]. In this case any tree-level masses the fermions obtain from, say, Higgs in the 10 could be drastically modified by threshold corrections (this possibility was in fact suggested in Ref. [13, 18]). The bad news is that tree-level relations such as $m_\tau = m_b$ may not scale as naively expected. The good news is that it may not be necessary to introduce extra Higgs like 126 solely to modify tree-level relations such as $m_e = m_d$. We are currently investigating how the model presented here fits into the $SO(10)$ framework [21].

One final comment we would like to make concerns the scale v_R where $SU(4) \times SU(2)_L \times SU(2)_R$ breaks to $SU(3) \times SU(2)_L \times U(1)_Y$. So long as $v_R \geq 10$ TeV, the mass formulas (40-42) are essentially independent of v_R . This means that if we were willing to give up the condition $g_L(v_R) = g_R(v_R)$ we could bring the scale v_R down to the several TeV range, as allowed by experiments [22]. However the largest top quark mass we can generate at the scale v_R is about 80 GeV, so we need a large hierarchy between v_L and v_R in order for it to scale by about the factor of two needed. In addition, since the physical neutrino masses are

suppressed by the see-saw mechanism, a low v_R would imply that the neutrino masses are close to their direct experimental upper bounds [23], and we would have to give up the oscillation solution to the solar neutrino problem. These last two observations suggest that v_R is indeed large, and $\geq 10^{12}$ GeV.

V. Conclusions.

We have presented a model for radiative fermion mass based on the gauge group $SU(4) \times SU(2)_L \times SU(2)_R$, with the fermion content of the Standard Model extended to include one sterile neutrino per generation. We are able to generate realistic fermion masses and mixings without a large hierarchy of coupling constants or extra family symmetries. The electron and muon neutrino masses lie in a range compatible with the MSW solution to the solar neutrino problem. The model demonstrates the potential importance of threshold corrections to tree-level mass relations in certain GUTs. The possibility of CP violation and baryogenesis in this model are being investigated.

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Appendix

In this appendix we would like to discuss the Higgs potential for the full-blown model of Sec. 4. In order to do so we must write down the most general $SU(4) \times SU(2)_L \times SU(2)_R$ invariant potential involving the fields $L_{i\alpha}$, $\Lambda_{i\alpha} \sim (\underline{4}, \underline{2}, \underline{1})$ and $R_{i\alpha}$, $T_{i\alpha} \sim (\underline{4}, \underline{1}, \underline{2})$. This potential will just be a long and complicated generalization of Eq. (18), but before we write it out explicitly, we first define the following 2×2 matrix fields

$$\begin{aligned}
X^i_j &= L^{i\alpha} L_{j\alpha} & Y^i_j &= \Lambda^{i\alpha} \Lambda_{j\alpha} & Z^i_j &= L^{i\alpha} \Lambda_{j\alpha} \\
\widetilde{X}^i_j &= L^i_\alpha L^\alpha_j & \widetilde{Y}^i_j &= \Lambda^i_\alpha \Lambda^\alpha_j & \widetilde{Z}^i_j &= L^i_\alpha \Lambda^\alpha_j \\
A^i_j &= R^{i\alpha} R_{j\alpha} & B^i_j &= T^{i\alpha} T_{j\alpha} & C^i_j &= R^{i\alpha} T_{j\alpha} \\
\widetilde{A}^i_j &= R^i_\alpha R^\alpha_j & \widetilde{B}^i_j &= T^i_\alpha T^\alpha_j & \widetilde{C}^i_j &= R^i_\alpha T^\alpha_j \\
H^i_j &= L^{i\alpha} R_{j\alpha} & I^i_j &= L^{i\alpha} T_{j\alpha} & J^i_j &= \Lambda^{i\alpha} T_{j\alpha} & K^i_j &= \Lambda^{i\alpha} R_{j\alpha} \\
\widetilde{H}^i_j &= L^i_\alpha R^\alpha_j & \widetilde{I}^i_j &= L^i_\alpha T^\alpha_j & \widetilde{J}^i_j &= \Lambda^i_\alpha T^\alpha_j & \widetilde{K}^i_j &= \Lambda^i_\alpha R^\alpha_j,
\end{aligned} \tag{54}$$

and as we have mentioned before, i is an $SU(2)$ index, α is the $SU(4)$ index, $\Psi^{i\alpha} = (\Psi_{i\alpha})^*$ and $\Psi_\alpha^i = \epsilon^{ij}\Psi_{j\alpha}$. As an example,

$$\widetilde{H}^i_j = \begin{pmatrix} L_d R^d + L_e R^e & -(L_d R^u + L_e R^\nu) \\ -(L_u R^d + L_\nu R^e) & L_u R^u + L_\nu R^\nu \end{pmatrix} \quad (55)$$

Using the fields defined in Eq. (54) we have

$$\begin{aligned} V(L, R, \Lambda, T) &= -2\mu_X^2 X_i^i - 2\mu_Y^2 Y_i^i - \frac{1}{2}\mu_Z^2 [Z_i^i + h.c.] \\ &+ \lambda_{XX1} X_i^i X_j^j + \lambda_{XX2} X_j^i X_i^j + \lambda_{XX3} X_j^i \widetilde{X}_i^j \\ &+ \lambda_{YY1} Y_i^i Y_j^j + \lambda_{YY2} Y_j^i Y_i^j + \lambda_{YY3} Y_j^i \widetilde{Y}_i^j \\ &+ \lambda_{XY1} X_i^i Y_j^j + \lambda_{XY2} X_j^i Y_i^j + \lambda_{XY3} X_j^i \widetilde{Y}_i^j \\ &+ \lambda_{XZ1} [X_i^i Z_j^j + h.c.] + \lambda_{XZ2} [X_j^i Z_i^j + h.c.] + \lambda_{XZ3} [X_j^i \widetilde{Z}_i^j + h.c.] \\ &+ \lambda_{YZ1} [Y_i^i Z_j^j + h.c.] + \lambda_{YZ2} [Y_j^i Z_i^j + h.c.] + \lambda_{YZ3} [Y_j^i \widetilde{Z}_i^j + h.c.] \\ &+ \lambda_{ZZ} Z_i^i Z_j^j + \lambda_{ZZ1} [Z_i^i Z_j^j + h.c.] + \lambda_{ZZ2} Z_j^i Z_i^j + \lambda_{ZZ3} [Z_j^i \widetilde{Z}_i^j + h.c.] \\ &- 2\mu_A^2 A_i^i - 2\mu_B^2 B_i^i - \frac{1}{2}\mu_C^2 [C_i^i + h.c.] \\ &+ \lambda_{AA1} A_i^i A_j^j + \lambda_{AA2} A_j^i A_i^j + \lambda_{AA3} A_j^i \widetilde{A}_i^j \\ &+ \lambda_{BB1} B_i^i B_j^j + \lambda_{BB2} B_j^i B_i^j + \lambda_{BB3} B_j^i \widetilde{B}_i^j \\ &+ \lambda_{AB1} A_i^i B_j^j + \lambda_{AB2} A_j^i B_i^j + \lambda_{AB3} A_j^i \widetilde{B}_i^j \\ &+ \lambda_{AC1} [A_i^i C_j^j + h.c.] + \lambda_{AC2} [A_j^i C_i^j + h.c.] + \lambda_{AC3} [A_j^i \widetilde{C}_i^j + h.c.] \\ &+ \lambda_{BC1} [B_i^i C_j^j + h.c.] + \lambda_{BC2} [B_j^i C_i^j + h.c.] + \lambda_{BC3} [B_j^i \widetilde{C}_i^j + h.c.] \\ &+ \lambda_{CC} C_i^i C_j^j + \lambda_{CC1} [C_i^i C_j^j + h.c.] + \lambda_{CC2} C_j^i C_i^j + \lambda_{CC3} [C_j^i \widetilde{C}_i^j + h.c.] \\ &+ \lambda_{HH1} X_i^i A_j^j + \lambda_{HH2} H_j^i H_i^j + \lambda_{HH3} [H_j^i \widetilde{H}_i^j + h.c.] \\ &+ \lambda_{II1} X_i^i B_j^j + \lambda_{II2} I_j^i I_i^j + \lambda_{II3} [I_j^i \widetilde{I}_i^j + h.c.] \\ &+ \lambda_{JJ1} Y_i^i B_j^j + \lambda_{JJ2} J_j^i J_i^j + \lambda_{JJ3} [J_j^i \widetilde{J}_i^j + h.c.] \\ &+ \lambda_{KK1} Y_i^i A_j^j + \lambda_{KK2} K_j^i K_i^j + \lambda_{KK3} [K_j^i \widetilde{K}_i^j + h.c.] \\ &+ \lambda_{HI1} [X_i^i C_j^j + h.c.] + \lambda_{HI2} [H_j^i I_i^j + h.c.] + \lambda_{HI3} [H_j^i \widetilde{I}_i^j + h.c.] \\ &+ \lambda_{JK1} [Y_i^i C_j^j + h.c.] + \lambda_{JK2} [J_j^i K_i^j + h.c.] + \lambda_{JK3} [J_j^i \widetilde{K}_i^j + h.c.] \\ &+ \lambda_{HK1} [Z_i^i A_j^j + h.c.] + \lambda_{HK2} [H_j^i K_i^j + h.c.] + \lambda_{HK3} [H_j^i \widetilde{K}_i^j + h.c.] \\ &+ \lambda_{IJ1} [Z_i^i B_j^j + h.c.] + \lambda_{IJ2} [I_j^i J_i^j + h.c.] + \lambda_{IJ3} [I_j^i \widetilde{J}_i^j + h.c.] \\ &+ \lambda_{HJ1} [Z_i^i C_j^j + h.c.] + \lambda_{HJ2} [H_j^i J_i^j + h.c.] + \lambda_{HJ3} [H_j^i \widetilde{J}_i^j + h.c.] \\ &+ \lambda_{IK1} [Z_i^i C_j^j + h.c.] + \lambda_{IK2} [I_j^i K_i^j + h.c.] + \lambda_{IK3} [I_j^i \widetilde{K}_i^j + h.c.] \quad (56) \end{aligned}$$

We now assume the vacuum expectation values

$$\begin{aligned} \langle L \rangle &= \begin{pmatrix} 0 & 0 & 0 & \frac{v_L c_L}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \end{pmatrix}; & \langle \Lambda \rangle &= \begin{pmatrix} 0 & 0 & 0 & \frac{v_L s_L e^{i\phi_L}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \end{pmatrix} \\ \langle R \rangle &= \begin{pmatrix} 0 & 0 & 0 & \frac{v_R c_R e^{i\delta}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \end{pmatrix}; & \langle T \rangle &= \begin{pmatrix} 0 & 0 & 0 & \frac{v_R s_R e^{i(\delta+\phi_R)}}{\sqrt{2}} \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \quad (57)$$

and look for extrema of the potential. Positivity of the Higgs masses will ensure that this extremum is at least a local minimum.

In order for the charged leptons to get a one-loop mass, we need L_e , Λ_e to mix with R_e , T_e . However, these are the Higgses that are eaten by the gauge bosons W_L , W_R , which can't mix at tree level in this model. Thus the only way the charged leptons can get a mass is if some of the Higgses don't get a vacuum expectation value. The choice we make is $\langle \Lambda \rangle = \langle T \rangle = 0$, *i.e.* $s_L = s_R = 0$ in Eq. (57). This choice also ensures that all the potentially CP violating phases ϕ_L , ϕ_R , δ are unphysical, and we set them equal to 0. We are then left with the following four equations that we need to satisfy in order to get the desired pattern of symmetry breaking:

$$(\lambda_{XX1} + \lambda_{XX2})v_L^2 + \frac{1}{2}(\lambda_{HH1} + \lambda_{HH2})v_R^2 = 2\mu_X^2 \quad (58)$$

$$\frac{1}{2}(\lambda_{HH1} + \lambda_{HH2})v_L^2 + (\lambda_{AA1} + \lambda_{AA2})v_R^2 = 2\mu_A^2 \quad (59)$$

$$(\lambda_{XZ1} + \lambda_{XZ2})v_L^2 + (\lambda_{HK1} + \lambda_{HK2})v_R^2 = 4\mu_Z^2 \quad (60)$$

$$(\lambda_{HI1} + \lambda_{HI2})v_L^2 + (\lambda_{AC1} + \lambda_{AC2})v_R^2 = 4\mu_C^2 \quad (61)$$

Eqs. (58,59) which are identical to Eqs. (22,23) of Sec. 3 determine the scales v_L, v_R . One choice of parameters consistent with the 'minimal fine-tuning hypothesis' [25, 26] is $\mu_X \sim v_L$, $\mu_A \sim v_R$ and $\lambda_{HH1} + \lambda_{HH2} = 0$. In which case we get

$$v_L^2 = \frac{2\mu_X^2}{\lambda_{XX1} + \lambda_{XX2}} \quad (62)$$

$$v_R^2 = \frac{2\mu_A^2}{\lambda_{AA1} + \lambda_{AA2}}, \quad (63)$$

with $\lambda_{AA1} + \lambda_{AA2}$, $\lambda_{XX1} + \lambda_{XX2} \sim 1$. Eqs. (60,61) ensure that $\langle \Lambda \rangle = \langle T \rangle = 0$. This also ensures that L_e , R_e don't mix with Λ_e , T_e . There will in general be one light neutral Higgs (corresponding to the Standard Model Higgs boson), with the rest having masses $\sim v_R$.

In order to simplify the calculation, we will adjust the Higgs parameters to ensure a similar condition for the other Higgs bosons *i.e.* no mixing between $L - R$ and $\Lambda - T$ sectors. The 4×4 Higgs mass matrices now break up into 2×2 blocks. Consider as an example the mass squared matrices for L_u , R_u , Λ_u , T_u :

$$M_{LR}^u = -\frac{\lambda_{HH2}}{2}v_R^2 \begin{pmatrix} 1 & -\epsilon \\ -\epsilon & \epsilon^2 \end{pmatrix}. \quad (64)$$

The masses and mixing angles are then

$$M_{LRu1}^2 = -\frac{\lambda_{HH2}}{2}v_R^2; \quad M_{LRu2}^2 = 0; \quad s_{LRu} = \epsilon, \quad (65)$$

where $\epsilon = v_L/v_R$. Since we are calculating in 'tHooft-Feynman gauge, we use the gauge boson mass for the mass of the Nambu-Goldstone boson. Thus, when calculating the contribution from Eq. (41) we use

$$M_{LRu2}^2 = \frac{g_S^2 v_R^2}{4} \quad (66)$$

Notice that we have no freedom to vary the mixing angle s_{LRu} . The mass squared matrix for Λ_u , T_u is

$$M_{\Lambda T}^u = \frac{v_R^2}{2} \begin{pmatrix} \lambda_{KK1} & \lambda_{HJ2}\epsilon \\ \lambda_{HJ2}\epsilon & \lambda_{AB1} + \lambda_{AB2} \end{pmatrix}, \quad (67)$$

with eigenvalues and mixing angles

$$M_{\Lambda Tu1}^2 = \lambda_{KK1} \frac{v_R^2}{2}; \quad M_{\Lambda Tu2}^2 = (\lambda_{AB1} + \lambda_{AB2}) \frac{v_R^2}{2}; \quad s_{\Lambda Tu} = \frac{\lambda_{HJ2}}{\lambda_{KK1} - (\lambda_{AB1} + \lambda_{AB2})} \epsilon. \quad (68)$$

Similarly the mass squared matrices for L_d , R_d , Λ_d , T_d are:

$$M_{LR}^d = \frac{v_R^2}{2} \begin{pmatrix} \lambda_{HH1} & 2\lambda_{HH3}\epsilon \\ 2\lambda_{HH3}\epsilon & 2(\lambda_{AA1} + \lambda_{AA3}) \end{pmatrix}, \quad (69)$$

with

$$M_{LRd1}^2 = \frac{\lambda_{HH1}}{2}v_R^2; \quad M_{LRd2}^2 = (\lambda_{AA1} + \lambda_{AA3})v_R^2; \quad s_{LRd} = \frac{2\lambda_{HH3}}{\lambda_{HH1} - 2(\lambda_{AA1} + \lambda_{AA3})} \epsilon. \quad (70)$$

and

$$M_{\Lambda T}^d = \frac{v_R^2}{2} \begin{pmatrix} \lambda_{KK1} & \lambda_{HJ3}\epsilon \\ \lambda_{HJ3}\epsilon & \lambda_{AB1} + \lambda_{AB3} \end{pmatrix}, \quad (71)$$

giving

$$M_{\Lambda T d1}^2 = \frac{\lambda_{KK1}}{2} v_R^2; \quad M_{\Lambda T d2}^2 = (\lambda_{AB1} + \lambda_{AB3}) \frac{v_R^2}{2}; \quad s_{\Lambda T d} = \frac{\lambda_{HJ3}}{\lambda_{KK1} - (\lambda_{AB1} + \lambda_{AB3})} \epsilon. \quad (72)$$

These are the values we use to calculate the contribution to the up and down type quark masses from Eq. (42). It would appear from Eq. (68) that we can make the mixing angle as large as we want by tuning $\lambda_{KK1} - (\lambda_{AB1} + \lambda_{AB2})$ to be small. However as the mass terms in the same equation show, this would make the Higgs degenerate in mass, and make the two terms in Eq. (42) cancel. Thus there is an upper limit to the masses we can get in this model. It is interesting to note that even if we saturate the coupling constants in this model to be largest they can be consistent with maintaining a perturbative theory (Yukawa couplings $\kappa \sim 3.5$, Higgs couplings $\lambda \sim 4$), the largest top quark mass we can get in this model is $m_t \sim 200$ GeV!

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